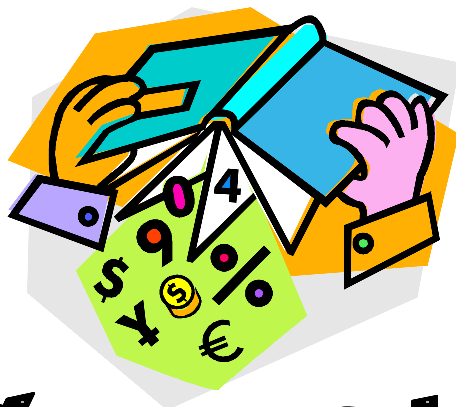
Mathematics:



"A Story of Units"

Parent Handbook

Grade 5
Module 3

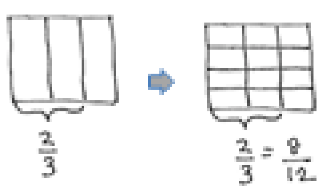
Grade 5 • Module 3

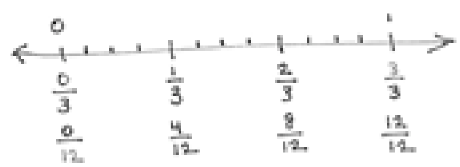
Addition and Subtraction of Fractions

OVERVIEW

In Module 3, students' understanding of addition and subtraction of fractions extends from earlier work with fraction equivalence and decimals. This module marks a significant shift away from the elementary grades' centrality of base ten units to the study and use of the full set of fractional units from Grade 5 forward, especially as applied to algebra.

In Topic A, students revisit the foundational Grade 4 standards addressing equivalence. When equivalent, fractions represent the same amount of area of a rectangle, the same point on the number line. These equivalencies can also be represented symbolically.





$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Furthermore, equivalence is evidenced when adding fractions with the same denominator. The sum may be decomposed into parts (or re-composed into an equal sum). For example:

$$\frac{2}{3} = \frac{1}{3} + \frac{1}{3}$$

$$\frac{7}{8} = \frac{3}{8} + \frac{3}{8} + \frac{1}{8}$$

$$\frac{6}{2} = \frac{2}{2} + \frac{2}{2} + \frac{2}{2} = 1 + 1 + 1 = 3$$

$$\frac{8}{5} = \frac{5}{5} + \frac{3}{5} = 1\frac{3}{5}$$

$$\frac{7}{3} = \frac{6}{3} + \frac{1}{3} = 2 \times \frac{3}{3} + \frac{1}{3} = 2 + \frac{1}{3} = 2\frac{1}{3}$$

This is also carrying forward work with decimal place value from Modules 1 and 2, confirming that like units can be composed and decomposed.

5 tenths + 7 tenths = 12 tenths = 1 and 2 tenths
5 eighths + 7 eighths = 12 eighths = 1 and 4 eighths

In Topic B, students move forward to see that fraction addition and subtraction is analogous to whole number addition and subtraction. Students add and subtract fractions with unlike denominators by replacing different fractional units with an equivalent fraction or like unit.

1 fourth + 2 thirds = 3 twelfths + 8 twelfths = 11 twelfths

$$\frac{1}{4}+\frac{2}{3}=\frac{3}{12}+\frac{8}{12}=\frac{11}{12}$$

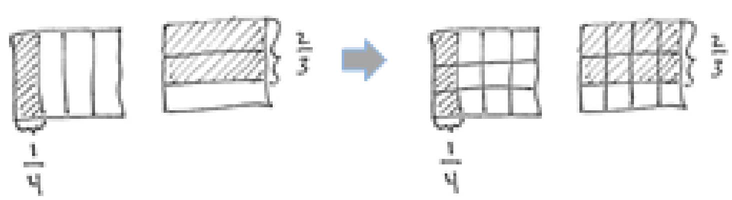
This is not a new concept but certainly a new level of complexity. Students have added equivalent or like units since kindergarten, adding frogs to frogs, ones to ones, tens to tens, etc.

1 boy + 2 girls = 1 child + 2 children = 3 children

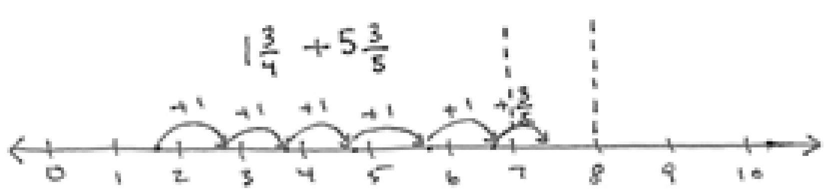
1 liter – 375 mL = 1,000 mL – 375 mL = 625 mL

Throughout the module, a concrete to pictorial to abstract approach is used to convey this simple concept. Topic A uses paper strips and number line diagrams to clearly show equivalence. After a brief concrete introduction with folding paper, Topic B primarily uses the rectangular fractional model because it is useful for creating smaller like units via partitioning (e.g., thirds and fourths are changed to twelfths to create equivalent fractions as in the diagram below.) In Topic C, students move away from the pictorial altogether as they are empowered to write equations clarified by the model.

$$\frac{1}{4}+\frac{2}{3}=\left(\frac{1\times 3}{4\times 3}\right)+\left(\frac{2\times 4}{3\times 4}\right)=\frac{3}{12}+\frac{8}{12}=\frac{11}{12}$$



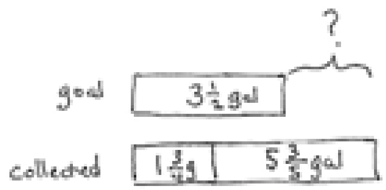
Topic C also uses the number line when adding and subtracting fractions greater than or equal to 1 so that students begin to see and manipulate fractions in relation to larger whole numbers and to each other. The number line takes fractions into the larger set of whole numbers. For example, “Between what two whole numbers will the sum of 1 $^3/_4$ and 5 $^3/_5$ lie?”



$$\_\_\_ < 1\frac{3}{4}+5\frac{3}{5} < \_\_\_$$

This leads to understanding of and skill with solving more interesting problems, often embedded within multi-step word problems:

Cristina and Matt's goal is to collect a total of 3 ½ gallons of sap from the maple trees. Cristina collected 1 ¾ gallons. Matt collected 5 ⅗ gallons. By how much did they beat their goal?



$$1\frac{3}{4}\text{ gal} + 5\frac{3}{5}\text{ gal} - 3\frac{1}{2}\text{gal} = 3 + \left(\frac{3 \times 5}{4 \times 5}\right) + \left(\frac{3 \times 4}{5 \times 4}\right) - \left(\frac{1 \times 10}{2 \times 10}\right)$$

$$= 3 + \frac{15}{20} + \frac{12}{20} - \frac{10}{20} = 3\ \frac{17}{20}\ \text{gal}$$

Cristina and Matt beat their goal by 3 $^{17}/_{20}$ gallons.

Word problems are part of every lesson. Students are encouraged to draw bar diagrams, which allow analysis of the same part–whole relationships they have worked with since Grade 1.

In Topic D, students strategize to solve multi-term problems and more intensely assess the reasonableness both of their solutions to word problems and their answers to fraction equations.

"I know my answer makes sense because the total amount of sap they collected is going to be about 7 and a half gallons. Then, when we subtract 3 gallons, that is about 4 and a half. Then, 1 half less than that is about 4. 3 $^{17}/_{20}$ is just a little less than 4."

**The sample questions/responses contained in this manual are straight from http://www.engageny.org/. They are provided to give some insight into the kinds of skills expected of students as the lesson is taught.

Terminology

New or Recently Introduced Terms

- Benchmark fraction (e.g., 1/2 is a benchmark fraction when comparing 1/3 and 3/5)
- Unlike denominators (e.g., 1/8 and 1/7)
- Like denominators (e.g., 1/8 and 5/8)

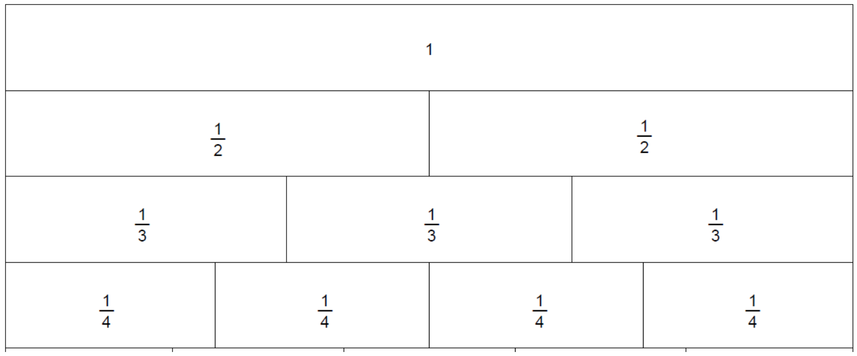
Familiar Terms and Symbols

- < , > , =
- Denominator (denotes the fractional unit: fifths in 3 fifths or the 5 in 3/5)
- Numerator (denotes the count of fractional units: 3 in 3 fifths or 3 in 3/5)
- Whole unit (e.g., any unit that is partitioned into smaller, equally sized fractional units)
- Fractional unit (e.g., the fifth unit in 3 fifths denoted by the denominator 5 in 3/5)
- Number sentence (e.g., "Three plus seven equals ten." Usually written as "3 + 7 = 10.")
- Meter, kilometer, centimeter, liter, kiloliter, gram, kilogram, feet, mile, yard, inch, gallon, quart, pint, cup, pound, ounce, hour, minute, second
- *More than halfway* and *less than halfway*
- *One tenth of* (e.g., 1/10 of 250)
- Fraction (e.g., 3 fifths or 3/5)
- Between (e.g., 1/2 is between 1/3 and 3/5)
- Fraction written in the largest possible unit (e.g., 3/6 = 1 x 3 / 2 x 3 = 1/2 or 1 three out of 2 threes = ½)
- Equivalent fraction (e.g., 3/5 = 6/10)
- Tenth (1/10 or 0.1)
- Hundredth (1/100 or 0.01)
- Fraction greater than or equal to 1 (e.g., 7/3, 3 1/2, an abbreviation for 3 + 1/2)

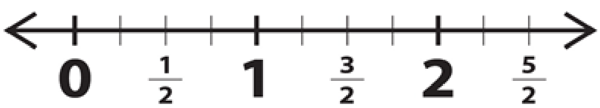
Suggested Tools and Representations

- Paper strips (for modeling equivalence)
- Number line (a variety of templates)
- Rectangular fraction model
- Fraction strips
- Tape diagrams (Also known as bar diagrams or bar models)

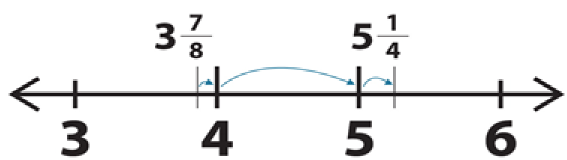
Fraction Strips: Fraction strips are tiles or strips that are proportionately sized to one whole so that students may physically make size comparisons and find equivalent amounts using different denominators.



Number Lines: The number line is showing fractions and improper fractions located between whole numbers. The “empty” number line initially shows whole numbers only without portions or increments between whole numbers indicated, so as to allow for students to approximate locations of fractions, improper fractions and/or mixed numbers (ie. 5¼).

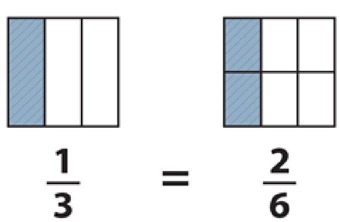


Example of a number line



Example of an “empty” number line

Rectangular Fractional Model: This rectangular fraction model allows students to begin with two whole rectangles of equal size which can be broken into thirds and sixths in order to find equivalent portions for fractions with unlike denominators.

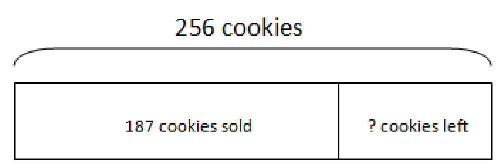


Example of a rectangular fraction model

Tape Diagram: Tape diagrams, also called bar models, are pictorial representations of relationships between quantities used to solve word problems. At the heart of a tape diagram is the idea of *forming units*. In fact, forming units to solve word problems is one of the most powerful examples of the unit theme and is particularly helpful for understanding fraction arithmetic. The tape diagram provides an essential bridge to algebra and is often called "pictorial algebra." There are two basic forms of the tape diagram model. The first form is sometimes called the part-whole model; it uses bar segments placed end-to-end (Grade 3 Example), while the second form, sometimes called the comparison model, uses two or more bars stacked in rows that are typically left justified. (Grade 5 Example depicts this model.)

Grade 3 Example: *Sarah baked 256 cookies. She sold some of them. 187 were left. How many did she sell?*

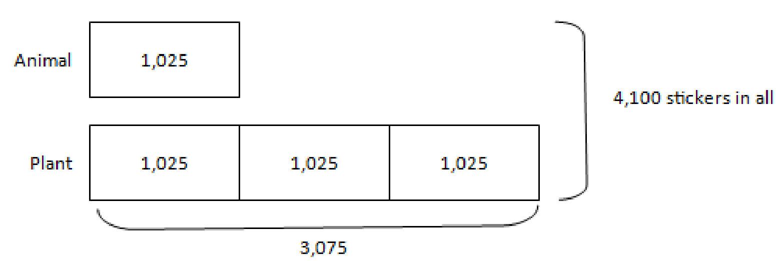
256 – 187 = _____



256 –187 = 69

Sarah sold 69 cookies.

Grade 5 Example: *Sam has 1,025 animal stickers. He has 3 times as many plant stickers as animal stickers. How many plant stickers does Sam have? How many stickers does Sam have altogether?*

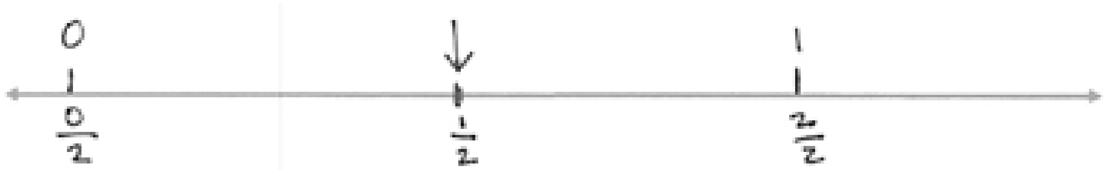


1. He has 3,075 plant stickers.
2. He has 4,100 stickers altogether.

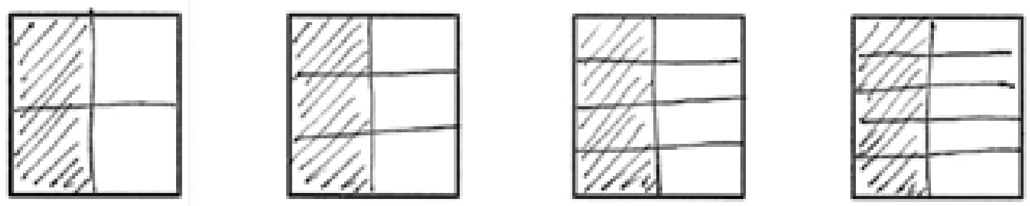
Lesson 1

Objective: Make equivalent fractions with the number line, the area model, and numbers

1. Use your folded paper strip to mark the points 0 and 1 above the number line and 0/2, 1/2, and 2/2 below it.



Draw one vertical line down the middle of each rectangle. Shade the left half of each. Partition with horizontal lines to show the equivalent fractions 2/4, 3/6, 4/8, 5/10. Use multiplication to show the change in the units.



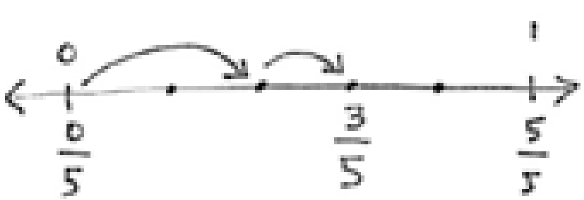
$\frac{1}{2} = \frac{1\times2}{2\times2} = \frac{2}{4}$ $\frac{1}{2} = \frac{1\times3}{2\times3} = \frac{3}{6}$ $\frac{1}{2} = \frac{1\times4}{2\times4} = \frac{4}{8}$ $\frac{1}{2} = \frac{1\times5}{2\times5} = \frac{5}{10}$

Lesson 2

Objective: Make equivalent fractions with sums of fractions with like denominators.

Show each expression on a number line. Solve.

a) $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$



Express each fraction as the sum of two or three equal fractional parts. Rewrite each as a multiplication equation. Show your answer to letter a on a number line.

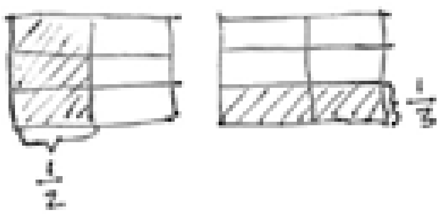
a) $\frac{6}{7} = \frac{3}{7} + \frac{3}{7} = 2 \times \frac{3}{7}$

Lesson 3

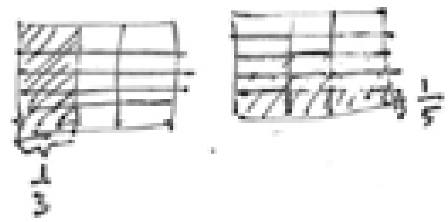
Objective: Add fractions with unlike units using the strategy of creating equivalent fractions.

For the following problems, draw a picture using the rectangular fraction model and write the answer. Simplify your answer.

a) $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$



b) $\frac{1}{3} + \frac{1}{5} = \frac{5}{15} + \frac{3}{15} = \frac{8}{15}$



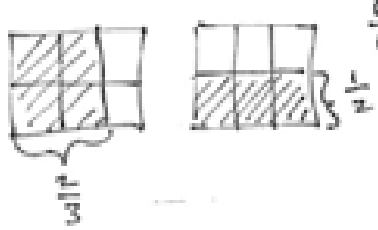
Lesson 4

Objective: Add fractions with sums between 1 and 2.

For the following problems, draw a picture using the rectangular fraction model and write the answer. When possible, write your answer as a mixed number.

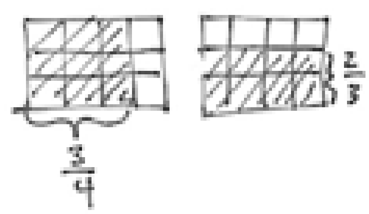
a) $\frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6} = \frac{7}{6} = 1\frac{1}{6}$

$\frac{6}{6}$ $\frac{1}{6}$



b) $\frac{3}{4} + \frac{2}{3} = \frac{9}{12} + \frac{8}{12} = \frac{17}{12} = 1\frac{5}{12}$

$\frac{12}{12}$ $\frac{5}{12}$

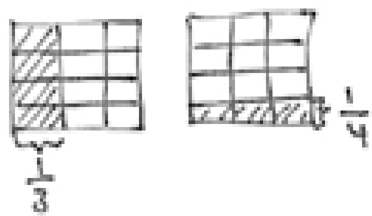


Lesson 5

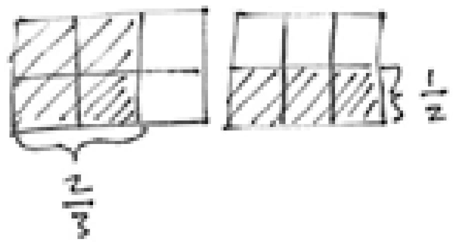
Objective: Subtract fractions with unlike units using the strategy of creating equivalent fractions.

For the following problems, draw a picture using the rectangular fraction model and write the answer. Simplify your answer.

a) $\frac{1}{3}-\frac{1}{4}=\frac{4}{12}-\frac{3}{12}=\frac{1}{12}$



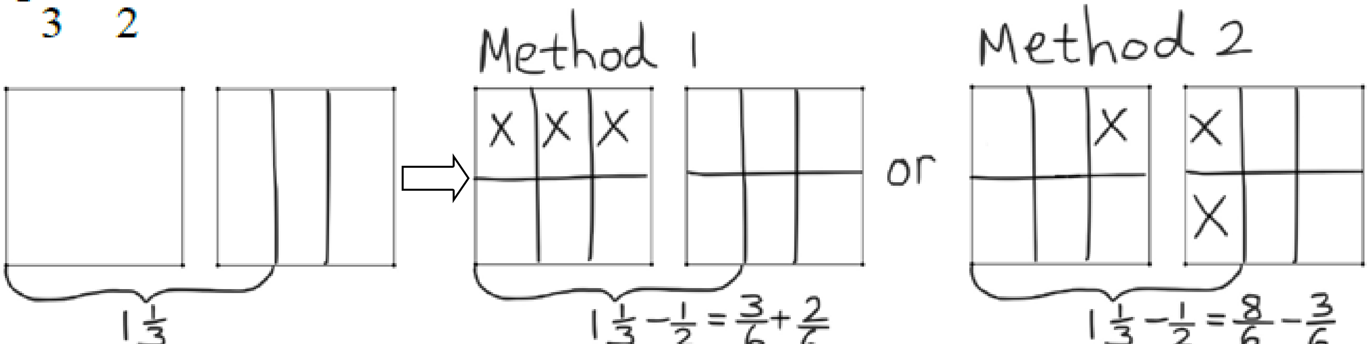
b) $\frac{2}{3}-\frac{1}{2}=\frac{4}{6}-\frac{3}{6}=\frac{1}{6}$



Lesson 6

Objective: Subtract fractions from numbers between 1 and 2.

$1\frac{1}{3}-\frac{1}{2}=$



Method 1:

$1\frac{1}{3}-\frac{1}{2}=\frac{3}{6}+\frac{2}{6}$

$\frac{6}{6}\ \ \frac{2}{6}\qquad =\frac{5}{6}$

Method 2:

$1\frac{1}{3}-\frac{1}{2}=\frac{8}{6}-\frac{3}{6}$

$=\frac{5}{6}$

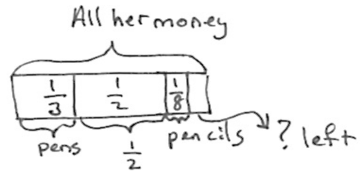
Renaming fractions using common denominators and/or breaking apart the mixed number (as in 1 1/3 into 6/6 and 2/6) enables students to subtract (cross out) the fraction using its equivalent (subtracting 1/2 is equal to subtracting 3/6). The brackets used in the models show the portions being used (in this case, the 1 1/3 out of the 2 wholes drawn).

Lesson 7

Objective: Solve two-step word problems.

Jing spent 1/3 of her money on a pack of pens, 1/2 of her money on a pack of markers, and 1/8 of her money on a pack of pencils. What fraction of her money is left?

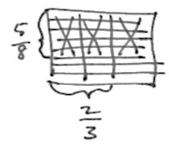
Two different methods of combing fractions/ making equivalents are shown.



$$1 - \frac{1}{3} - \frac{1}{2} - \frac{1}{8}$$

$$= \frac{2}{3} - \frac{1}{2} - \frac{1}{8}$$

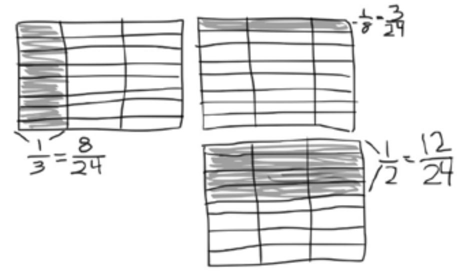
$$= \frac{2}{3} - \frac{5}{8}$$



$$= \frac{16}{24} - \frac{15}{24}$$

$$= \frac{1}{24}$$

Jing had $\frac{1}{24}$ of her money left.



$$\frac{1}{3} + \frac{1}{2} + \frac{1}{8}$$

$$= \frac{8}{24} + \frac{12}{24} + \frac{3}{24}$$

$$= \frac{23}{24}$$

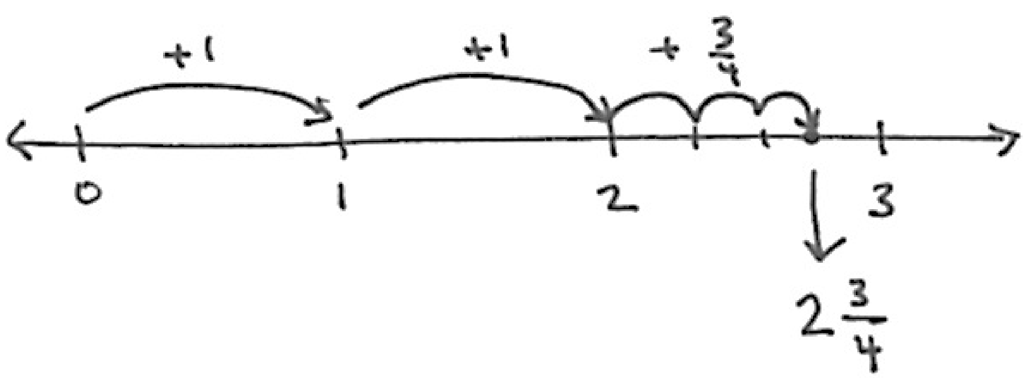
$$\frac{24}{24} - \frac{23}{24} = \frac{1}{24}$$

Jing had $\frac{1}{24}$ of her money left.

Lesson 8

Objective: Add fractions to and subtract fractions from whole numbers using equivalence and the number line as strategies.

1 + 1 3/4



Lesson 9

Objective: Add fractions making like units numerically.

First make like units. Then add.

a) $\frac{3}{4} + \frac{1}{7} = \left(\frac{3 \times 7}{4 \times 7}\right) + \left(\frac{1 \times 4}{7 \times 4}\right)$

$= \frac{21}{28} + \frac{4}{28}$

$= \frac{25}{28}$

Jackie brought 3/4 of a gallon of iced tea to the party. Bill brought 7/8 of a gallon of iced tea to the same party. How much iced tea did Jackie and Bill bring to the party?

$\frac{3}{4} + \frac{7}{8} = \left(\frac{3}{4} \times \frac{4}{4}\right) + \left(\frac{7}{8} \times \frac{2}{2}\right)$

$= \frac{12}{16} + \frac{14}{16}$

$= \frac{26}{16}$

$= \frac{16}{16} + \frac{10}{16} \Rightarrow 1\frac{10}{16}$ gallons or $1\frac{5}{8}$ gallons

Jackie and Bill brought $1\frac{5}{8}$ gallons of iced tea.

Lesson 10

Objective: Add fractions with sums greater than 2.

Add.

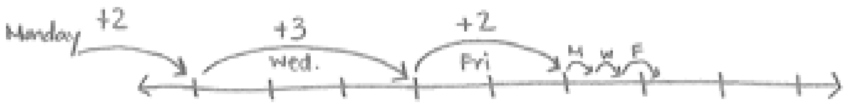
a) $2\frac{1}{4} + 1\frac{1}{5} = 3 + \frac{1}{4} + \frac{1}{5}$

$= 3 + \left(\frac{1}{4} \times \frac{5}{5}\right) + \left(\frac{1}{5} \times \frac{4}{4}\right)$

$= 3\frac{5}{20} + \frac{4}{20}$

$= 3\frac{9}{20}$

Erin jogged 2 1/4 miles on Monday. Wednesday she jogged 3 1/3 miles, and on Friday she jogged 2 2/3 miles. How far did Erin jog altogether?



$7\frac{1}{4} + \left(\frac{1}{3} + \frac{2}{3}\right) = 8\frac{1}{4}$

Erin jogged $8\frac{1}{4}$ miles altogether.

Lesson 11

Objective: Subtract fractions making like units numerically.

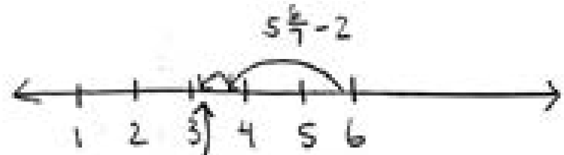
1) Generate equivalent fractions to get the same unit, then subtract.

g) $5\frac{6}{7} - 2\frac{2}{3} = 3\frac{6}{7} - \frac{2}{3}$

$= 3\frac{18}{21} - \frac{14}{21}$

$= 3\frac{4}{21}$

h) Draw a number line to show your answer to (g) is reasonable.



Lesson 12

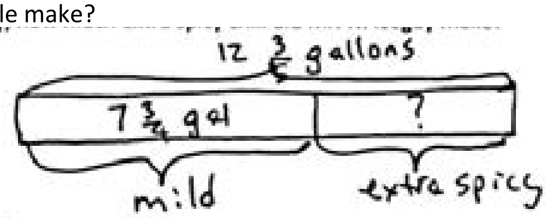
Objective: Subtract fractions greater than or equal to one.

a) $3\frac{1}{5} - 2\frac{1}{4} = \frac{3}{4} + \frac{1}{5}$

$3 \quad \frac{1}{5} \qquad = \frac{15}{20} + \frac{4}{20}$

$= \frac{19}{20}$

Neville mixed up 12 3/5 gallons of chili for a party. If 7 3/4 gallons of chili was mild, and the rest was extra spicy, how much extra spicy chili did Neville make?



$12\frac{3}{5} - 7\frac{3}{4} =$

$5\frac{3}{5} - \frac{3}{4} = 4\frac{8}{5} - \frac{3}{4} = 4\frac{32}{20} - \frac{15}{20} = 4\frac{17}{20}$

Mr. N. Icegy made $4\frac{17}{20}$ gallons.

Lesson 13

Objective: Use fraction benchmark numbers to assess reasonableness of addition and subtraction equations.

1. Are the following greater than or less than 1? Circle the correct answer.

a) $\frac{1}{2}+\frac{2}{7}$ greater than 1 less than 1

b) $\frac{5}{8}+\frac{3}{5}$ greater than 1 less than 1

4. Is it true that $4\frac{3}{5}-3\frac{2}{3}=1+\frac{3}{5}+\frac{2}{3}$? Prove your answer.

$$4\frac{3}{5}-3\frac{2}{3}=1\frac{3}{5}-\frac{2}{3}$$

$$=1+\frac{3}{5}-\frac{2}{3}$$

No! It's not true! It's $\frac{2}{3}$ less not more.

Lesson 14

Objective: Strategize to solve multi-term problems.

1. Rearrange the terms so that you can add or subtract mentally, then solve.

a) $\frac{1}{4}+2\frac{2}{3}+\frac{7}{4}+\frac{1}{3}$

$$\left(\frac{1}{4}+\frac{7}{4}\right)+\left(2\frac{2}{3}+\frac{1}{3}\right)=$$

$$2 + 3 = 5$$

b) $2\frac{3}{5}-\frac{3}{4}+\frac{2}{5}$

$$\left(2\frac{3}{5}+\frac{2}{5}\right)-\frac{3}{4}=$$

$$3-\frac{3}{4}=2\frac{1}{4}$$

2. Fill in the blank to make the statement true.

a) $11\frac{2}{5}-3\frac{2}{3}-\frac{11}{3}=$ 4 1/15

$$11\frac{2}{5}-6-\frac{4}{3}$$

$$=5\frac{2}{5}-1\frac{1}{3}$$

$$=4\frac{2}{5}-\frac{1}{3}=4\frac{6}{15}-\frac{5}{15}=4\frac{1}{15}$$

b) $11\frac{7}{8}+3\frac{1}{5}-$ 3/40 $=15$

$$14\frac{7}{8}+\frac{1}{5}$$

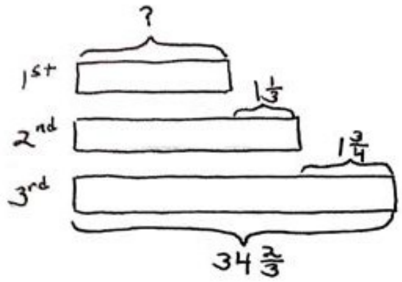
$$14\frac{35}{40}+\frac{8}{40}=$$

$$14\frac{43}{40}=15\frac{3}{40}-\frac{3}{40}=15$$

Lesson 15

Objective: Solve multi-step word problems; assess reasonableness of solutions using benchmark numbers.

In a race, the second place finisher crossed the finish line 1 1/3 minutes after the winner. The third place finisher was 1 3/4 minutes behind the second place finisher. The third place finisher took 34 2/3 minutes. How long did the winner take?



$$34\tfrac{2}{3} - 1\tfrac{3}{4} = 33\tfrac{2}{3} - \tfrac{3}{4}$$
$$= 33\tfrac{8}{12} - \tfrac{9}{12}$$
$$= 32\tfrac{20}{12} - \tfrac{9}{12}$$
$$= 32\tfrac{11}{12}$$
$$32\tfrac{11}{12} - 1\tfrac{1}{3} = 31\tfrac{11}{12} - \tfrac{1}{3}$$
$$= 31\tfrac{11}{12} - \tfrac{4}{12}$$
$$= 31\tfrac{7}{12}$$
$$31\tfrac{7}{12}\text{ min} = 31\tfrac{35}{60}\text{ min} = 31\text{ min } 35\text{s}$$

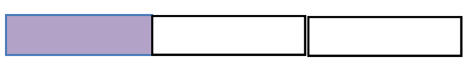
The 1st place time was 31 min 35s.

Lesson 16

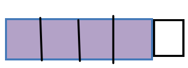
Objective: Explore part to whole relationships.

1. Draw the following ribbons. When finished, compare your work to your partner's.

   a) 1 ribbon. The piece shown below is only 1/3 of the whole. Complete the drawing to show the whole piece of ribbon.



   b) 1 ribbon. The piece shown below is 4/5 of the whole. Complete the drawing to show the whole piece of ribbon.



   c) 2 ribbons, A and B. One third of A is equal to all of B. Draw a picture of the ribbons.